Reduced Rank Fast-time STAP using the Multistage Wiener Filter in Multichannel SAR

Luke Rosenberg[*†]

[*]University of Adelaide, Australia
[†] Defence, Science and Technology Organisation, Australia
e-mail: `luker@eleceng.adelaide.edu.au`

Abstract: *Large regions of a Synthetic Aperture Radar (SAR) image can potentially be destroyed by an airborne broadband jammer. Jammer components include both the direct-path and multipath reflections from the ground, known as hot-clutter (HC) or terrain scattered interference. Using multiple antennas on a SAR provides spatial degrees of freedom and allows for beamforming to partially reject the interference components. Previous studies have shown that derivative constraints when combined with fast-time taps can provide improved HC suppression while maintaining a reasonable SAR image. This approach however requires an expensive matrix inverse and may not be implementable in real time. This paper therefore presents a fast-time Space Time Adaptive Processing (STAP) algorithm using a constrained Multistage Wiener Filter (MWF).*

1. Introduction

The goal of interference suppression for SAR is to successfully suppress the unwanted signals while not significantly effecting the image quality by blurring, reducing the resolution or raising the sidelobe level. This can be hard to achieve in practice, especially if the interference is non-stationary and the training statistics change from pulse to pulse, causing traditional slow-time STAP techniques be ineffective, [1]. Therefore adapting within each pulse is required by exploiting spatial/fast-time STAP. This offers the advantage of exploiting the coherency between the direct-path jammer and other HC signals to provide improved interference rejection but instead causes secondary modulations during image formation. In previous work, the use of derivative constraints to reduce potential signal suppression has shown to be an effective compromise to reduce the interference without compromising the target's range profile, [2].

Reduced rank techniques work to reduce the rank associated with the interference. Many of the methods in the literature promise performance near or better then their full rank counterparts but with reduced sample support and computation. The technique used in this paper is known as the MWF, [3] and provides a faster rank reduction using a nested chain of traditional Wiener filter stages. It is based on the beamspace generalised sidelobe canceller, [2] with weights now estimated at each stage to maximise the energy between the main and reference beams. This method also does not need an eigenvector decomposition or large covariance matrix inversion which makes it more suitable for real world implementation. Derivative constraints have also been applied to this algorithm by [4], who presented a very basic derivation. The new contribution in this paper is to present a simplified description of the MWF with modifications to use arbitrary constraints and fast-time taps and apply it to the problem of hot-clutter suppression in multichannel SAR.

2. Signal model

The total received signal at the SAR, $x_n(\cdot)$ includes the total ground return, interference from the direct-path and ground reflected path (hot-clutter) and receiver noise.
The bistatic jammer model is formed by the superposition of the direct path and K hot-clutter patches within a given area,

$$z_n(t_l,u) = \sum_{k=0}^{K} b_k J(t_l - \overline{\tau}_{n,k}(t_l,u)) \exp[-j\omega_c \overline{\tau}_{n,k}(t_l,u)] \exp[-j\omega_{d,k} t_l] \tag{1}$$

where (t_l,u) represents the l^{th} fast-time sample within a pulse for $l = 1 \ldots L$ and the SAR position respectively, $J(\cdot)$ is the jamming signal waveform, $\overline{\tau}_{n,k}(\cdot)$ is the bistatic delay for the k^{th} patch, $\omega_{d,k}$ is the fast-time doppler frequency and b_k is the relative magnitude between the direct-path and hot-clutter signals. The zero index refers to the direct-path with $b_0 = 1$.
Realisations of the jammer signal $J(\cdot)$ can be generated by an eigen-decomposition of the jammer auto-covariance, $r_J(\tau) = \sigma_J^2 \operatorname{sinc}(B\tau)$ with bandwidth, B and power level, σ_J^2. The relative scattering magnitude is determined by a physically based model for the multipath scattering, [5]. It uses a rough surface to define the scattering distribution between the SAR and an airborne jammer. The coefficients, $b_k = \rho B_k$ for $k > 1$ are formed with a hot-clutter scaling factor ρ, relative to the direct-path and a random amplitude B_k, determined from the scattering model.

3. Fast-time STAP

The conventional fast-time STAP output with $\tilde{L} \ll L$ fast-time taps is determined by the following convolution,

$$x_{\text{fs}}(t_l,u) = \mathbf{S}^H(u)\mathbf{X}(t_l,u) \tag{2}$$

where $\mathbf{S}(\cdot)$ is the space/fast-time steering vector and $\mathbf{X}(\cdot)$ is the fast-time received data vector. If there are N antenna elements, the received data signal can be stacked twice with the reference antenna at the centre of the array to give,

$$\mathbf{x}(t_l,u) = \left[x_{-(N-1)/2}(t_l,u), \ldots, x_{(N-1)/2}(t_l,u)\right]^T \in \mathcal{C}^{N\times 1} \tag{3}$$

$$\mathbf{X}(t_l,u) = \left[\mathbf{x}^T(t_l,u), \mathbf{x}^T(t_{l+1},u), \ldots, \mathbf{x}^T\left(t_{l+\tilde{L}-1},u\right)\right]^T \in \mathcal{C}^{\tilde{L}N\times 1} \tag{4}$$

with data components for the final $\tilde{L}$ taps set to zero. The spatial steering model for the n^{th} channel is given by,

$$s_n(u) = \exp\left[j\frac{\omega_c}{c} d_n \sin[\theta(u)]\right] \tag{5}$$

where ω_c is the carrier frequency, c is the speed of light, $d_n = n\lambda_c/2$ is the antenna offset from the array phase centre with wavelength λ_c and $\theta(u)$ is the steering angle relative to the centre of the imaging patch. The spatial steering vector, $\mathbf{s}(u)$ is then formed similarly to Equation 3. The fast-time component of the steering vector post range processing is given by,

$$g_q = \operatorname{sinc}[B(q-1)\Delta_t], \quad q = 1 \ldots \tilde{L} \tag{6}$$

where the fast-time sample rate, Δ_t is oversampled by a factor of two to provide increased correlation and effective fast-time filtering. It can be stacked to give the fast-time steering vector,

$$\mathbf{g} = \left[1, \operatorname{sinc}[B\Delta_t], \ldots, \operatorname{sinc}\left[B(\tilde{L}-1)\Delta_t\right]\right]^T \in \mathcal{C}^{\tilde{L}\times 1} \tag{7}$$

and the space/fast-time steering vector formed by the Kronecker product of the temporal and spatial steering vectors,

$$\mathbf{S}(u) = \mathbf{g} \otimes \mathbf{s}(u) \in \mathcal{C}^{\tilde{L}N\times 1} \tag{8}$$

Fast-time STAP then involves replacing the steering vector with a weight, $\mathbf{W}(u)$ designed to remove the interference from the received data.

4. Multistage Wiener Filter

The space/fast-time constrained MWF of order P is formed from P filter stages as shown in Figure 1, where null$[\mathbf{W}_\mathrm{d}(u)]$ represents the nullspace of $\mathbf{W}_\mathrm{d}(u)$.

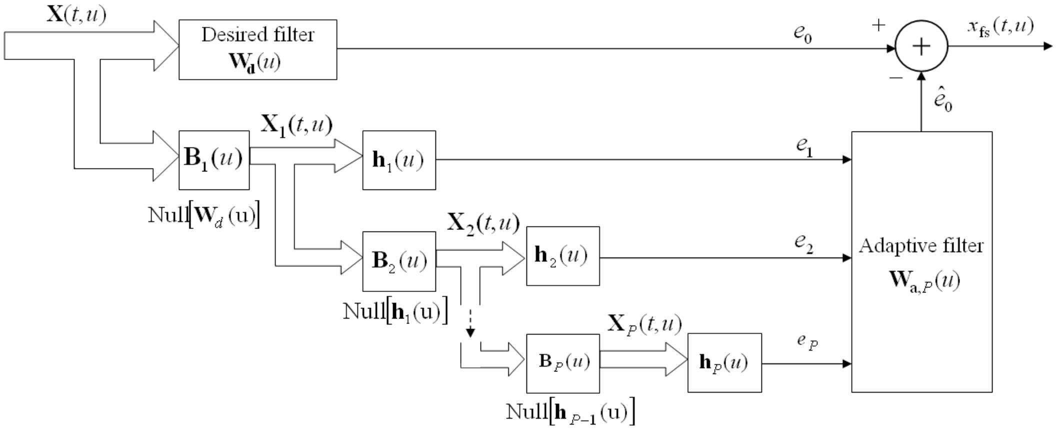

Figure 1: P stage Wiener filter

The overall MWF weight vector for the p^{th} stage is determined by,

$$\mathbf{W}(u) = \mathbf{W}_\mathrm{d}(u) - \mathbf{L}_p(u)\mathbf{W}_{\mathrm{a},p}(u) \in \mathcal{C}^{\tilde{L}N\times 1} \tag{9}$$

where the desired signal is defined by,

$$\mathbf{W}_\mathrm{d}(u) = \mathbf{C}(u)\left[\mathbf{C}^H(u)\mathbf{C}(u)\right]^{-1}\mathbf{D} \in \mathcal{C}^{\tilde{L}N\times 1} \tag{10}$$

with $\mathbf{C}(u)$ containing the N_con adaptive constraints, usually expressed as a function of the steering vector with desired response, $\mathbf{D}$. To successfully remove the desired signal in the reference beam, the first stage blocking matrix must to be orthogonal to the constraint matrix, $\mathbf{B}_1^H(u)\mathbf{C}(u) = 0$. A general method for the blocking matrix design has been presented in [2]. The sequential vector, $\mathbf{L}_p(\cdot)$ is defined by,

$$\mathbf{L}_p(u) = [\mathbf{B}_1(u)\mathbf{h}_1(u),\ \mathbf{B}_1(u)\mathbf{B}_2(u)\mathbf{h}_2(u),\ldots,\mathbf{B}_1(u)\mathbf{B}_2(u)\cdots\mathbf{B}_p(u)\mathbf{h}_p(u)] \in \mathcal{C}^{\tilde{L}N\times p}$$

with the rank one space/fast-time basis vectors, $\mathbf{h}_p(\cdot)$ designed to maximise the cross-correlation energy between e_p and e_{p-1},

$$\mathbf{h}_p(u) = \frac{\mathbf{r}_{x_p,e_{p-1}}}{\sqrt{\mathbf{r}^H_{x_p,e_{p-1}}(u)\mathbf{r}_{x_p,e_{p-1}}(u)}} \in \mathcal{C}^{\tilde{L}(N-N_\mathrm{con})\times 1} \tag{11}$$

with reference covariance and cross covariance,

$$\mathbf{R}_{x_p}(u) = \mathbf{B}_p^H(u)\mathbf{R}_{x_{p-1}}(u)\mathbf{B}_p(u) \in \mathcal{C}^{\tilde{L}(N-N_\mathrm{con})\times\tilde{L}(N-N_\mathrm{con})}, \tag{12}$$

$$\mathbf{r}_{x_p,e_{p-1}}(u) = \mathbf{B}_p^H(u)\mathbf{R}_{x_{p-1}}(u)\mathbf{h}_{p-1}(u) \in \mathcal{C}^{\tilde{L}(N-N_\mathrm{con})\times 1} \tag{13}$$

where $\mathbf{h}_0(u) = \mathbf{W}_\mathrm{d}(u)$, $\mathbf{R}_{x_0}(u) = \hat{\mathbf{R}}_Z(u)$ and the blocking matrices for $p > 1$,

$$\mathbf{B}_p(u) = \mathbf{I}_{\tilde{L}(N-N_\mathrm{con})} - \mathbf{h}_{p-1}(u)\mathbf{h}^H_{p-1}(u) \in \mathcal{C}^{\tilde{L}(N-N_\mathrm{con})\times\tilde{L}(N-N_\mathrm{con})} \tag{14}$$

The interference plus noise covariance matrix is determined by the sample matrix estimate,

$$\hat{\mathbf{R}}_Z(u) = \frac{1}{L_t}\sum_{l=1}^{L_t}\mathbf{Z}(t_l,u)\mathbf{Z}^H(t_l,u) \in \mathcal{C}^{\tilde{L}N\times\tilde{L}N} \tag{15}$$

where $\mathbf{Z}$ is determined as in Equation 4. The size of the p^{th} order space/fast-time weight vector is then determined by the MWF order and to take advantage of the fast-time taps, should be of the order of $\tilde{L}$. A diagonal loading level of η is also included prior to the matrix inverse to improve the robustness of the adaption,

$$\mathbf{W}_{\mathrm{a},p}(u) = \left[\mathbf{L}_p^H(u)\hat{\mathbf{R}}_Z(u)\mathbf{L}_p(u) + \eta\mathbf{I}_p\right]^{-1}\mathbf{L}_p^H(u)\hat{\mathbf{R}}_Z(u)\mathbf{W}_\mathrm{d}(u) \in \mathcal{C}^{p\times 1} \tag{16}$$

5. Simulated Results

The simulation is at X-band with $f_c = 10$GHz, $B = 0.3$GHz, $\sigma_J^2 = 80$dB and $\rho = 0.6$ and the jammer is incident in the SAR mainbeam. There are $N = 5$ spatial channels, $M = 100$ pulses, $L = 250$ range bins, $K = 200$ hot-clutter patches and the covariance matrix is estimated over $L_t = 3\tilde{L}N$ range bins. Both the Minimum Variance Distortionless Response (MVDR) and first order derivative constraints are tested with the latter constraint defined by setting the spatial constraint matrix, $\mathbf{c}(u)$ and desired vector, $\mathbf{d}$ to,

$$\mathbf{c}(u) = \left[\mathbf{s}(u), \frac{\partial \mathbf{s}(u)}{\partial \theta(u)}\right] \quad ; \qquad \mathbf{d} = [1, 0]^T \tag{17}$$

and the MVDR constraint defined with just the steering vector. The spatial constraints are then related to the space/fast-time versions by,

$$\mathbf{C}(u) = \mathbf{I}_{\tilde{L}} \otimes \mathbf{c}(u) \in \mathcal{C}^{\tilde{L}N \times \tilde{L}N_{\mathrm{con}}} \quad ; \qquad \mathbf{D} = \mathbf{g} \otimes \mathbf{d} \in \mathcal{C}^{\tilde{L}N_{\mathrm{con}} \times 1} \tag{18}$$

The adaptive performance is measured by the Signal Distortion Ratio (SDR) which is a measure of the signal power of the adapted image relative to an ideal image with no interference present. With no adaption, the conventional SDR is 3.8dB and the best full rank results are found using the method in [2]. For this simulation, the full rank MVDR result has an SDR of 6.5dB and for derivative constraints, the SDR is 7.1dB. Figure 2 shows the simulated results with both the filter order and diagonal loading level varied with $\tilde{L} = 15$ fast-time taps. It takes an order of 14 before the MVDR constrained filter behaves like the full rank case. In contrast, the derivative constraint results show that a small order of 5 can meet the full rank case without diagonal loading! This is huge difference of 11 filter orders and demonstrates the superiority of using the derivative constraints with the MWF.

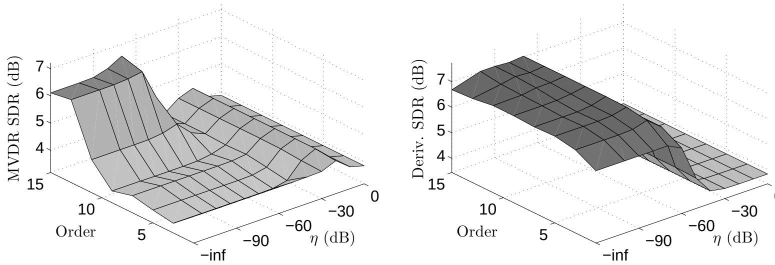

Figure 2: SDR for MVDR (left) and derivative constraints (right) with varying: order, η

6. Conclusion

This paper has demonstrated how the MWF can be used for hot-clutter rejection using fast-time STAP with constraints. Simulation results showed that combining derivative constraints with the MWF offers results matching the full-rank case with a significantly smaller filter size.

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